Calculus 1 Name______________________________

Review – Implicit Diff. and Related Rates (2) Block ______Date_________________

1) Find $\frac{dy}{dx}$ and the equation of the tangent line at the given point.

$$5x^2y^3+5x-6y=18xy \quad (3,1)$$

$5x^2y^3+5x-6y=18xy$

$10xy^3+5x^23y^2\frac{dy}{dx}+5-6\frac{dy}{dx}=18y+18x\frac{dy}{dx}$

$5x^23y^2\frac{dy}{dx}-6\frac{dy}{dx}-18x\frac{dy}{dx}=-10xy^3-5+18y$

$\frac{dy}{dx}\left(5x^23y^2-6-18x\right)=-10xy^3-5+18y$

$\boxed{\frac{dy}{dx}=\frac{-10xy^3-5+18y}{5x^23y^2-6-18x}}$

Plug in (3,1)

$\frac{dy}{dx}=\frac{-10(3)(1)^3-5+18(1)}{5(3)^23(1)^2-6-18(3)}$

$\frac{dy}{dx}=-\frac{17}{75}$

$y=mx+b$

$1=\left(-\frac{17}{75}\right)3+b$

$\frac{42}{25}=b$

$y=-\frac{17}{75}x+\frac{42}{25}$

or $\boxed{y-1=\frac{-17}{75}(x-3)}$

2) Find $\frac{dy}{dx}$ the derivative of $4x+3x^2y^4=30$.

$4+6xy^4+3x^24y^3\frac{dy}{dx}=0$

$3x^24y^3\frac{dy}{dx}=-4-6xy^4$

$\frac{dy}{dx}=\frac{-4-6xy^4}{12x^2y^3}$

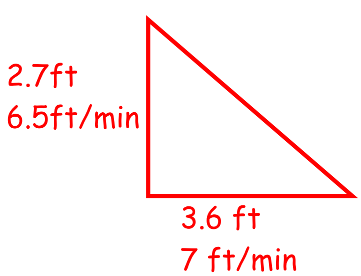
$\boxed{\frac{dy}{dx}=\frac{-2-3xy^4}{6x^2y^3}}$

3) Find the derivative of y with respect to x

$$4\sin\left(3x^3y\right)+5xy-7y^4=35.$$

$\boxed{4\cos\left(3x^3y\right)\left(9x^2y+3x^3\frac{dy}{dx}\right)+5y+5x\frac{dy}{dx}-28y^3\frac{dy}{dx}=0}$

4) A right triangle has legs with lengths of 2.7ft and 3.6ft. The longer leg is increasing at a rate 7ft/min and the shorter side is increasing at a rate of 6.5 ft/min. How fast is the hypotenuse changing at this instant? How fast is the angle between the shorter side and the hypotenuse changing?



$a^2+b^2=c^2$

$3.6^2+2.7^2=c^2$

$4.5=c$

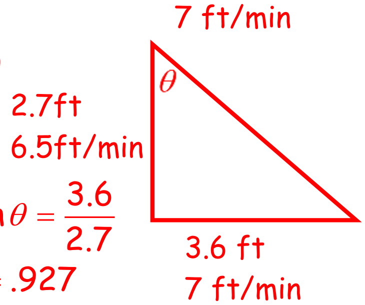
$a^2+b^2=c^2$

$2a\frac{da}{dt}+2b\frac{db}{dt}=2c\frac{dc}{dt}$

$2(3.6)(7)+2(2.7)(6.5)=2(4.5)\frac{dc}{dt}$

$\boxed{9.5\frac{ft}{min}=\frac{dc}{dt}}$

b)



$\tan\theta=\frac{3.6}{2.7}$

$\theta=.927$

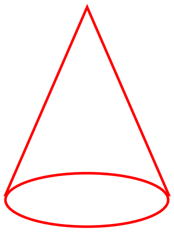
$\tan\theta=\frac{x}{y}$

$\sec^2\theta\frac{d\theta}{dt}=\frac{y\frac{dx}{dt}-x\frac{dy}{dt}}{y^2}$

$\sec^2(.927)\frac{d\theta}{dt}=\frac{2.7(7)-(3.6)(6.5)}{(2.7)^2}$

$\boxed{\frac{d\theta}{dt}=-.222\frac{rad}{min}}$

5) Sand is being poured at a rate of 7.6 cubic centimeters per second and is forming an conical pile. The height of the pile is always a third of the radius. Determine how fast the height is changing at the instant the height is 5 cm. Determine the rate of change of the radius at that same instant.



$$\frac{dV}{dt} = 7.6\frac{cm^3}{sec}$$

$$h = \frac{1}{3}r$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi (3h)^2 h$$

$$V = \frac{1}{3}\pi (3)^2 h^3$$

$$\frac{dV}{dt} = \frac{1}{3}\pi (3)^2 3h^2 \frac{dh}{dt}$$

$$7.6 = \frac{1}{3}\pi (3)^2 3(5)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.01075\frac{cm}{sec}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi r^2 \left(\frac{1}{3}r\right)$$

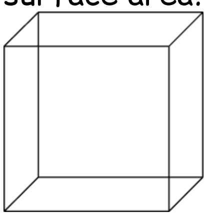
$$V = \frac{1}{3}\pi \frac{1}{3} r^3$$

$$\frac{dV}{dt} = \frac{1}{3}\pi \frac{1}{3} 3r^2 \frac{dr}{dt}$$

$$7.6 = \frac{1}{3}\pi \frac{1}{3} 3(15)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = 0.0323\frac{cm}{sec}$$

6) A perfect cube has side length of 14 in and the sides are decreasing at a rate of .5 in per second. What is the rate of change of the volume of this cube? What is the rate of change of the surface area?



$$s = 14in$$

$$\frac{ds}{dt} = -.5\frac{in}{sec}$$

$$V = s^3$$

$$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$$

$$\frac{dV}{dt} = 3(14)^2(-.5)$$

$$\frac{dV}{dt} = -294\frac{in^3}{sec}$$

$$SA = 6s^2$$

$$\frac{dSA}{dt} = 12s\frac{ds}{dt}$$

$$\frac{dSA}{dt} = 12(14)(-.5)$$

$$\frac{dSA}{dt} = -84\frac{in^2}{sec}$$